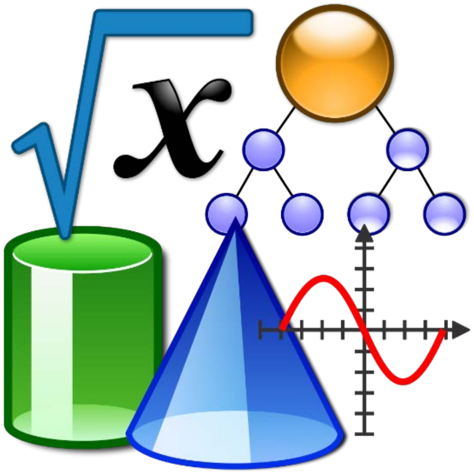
NPS Learning in Place

Geometry



Name:________________________ School:________________ Teacher: _______________

April 27 – May 15

Week 1	- **Reasoning Lines and Transformations**
Week 2	- **Parallel Lines**
Week 3	- **Congruent and Similar Triangles**

Week 1

Geometry RC1: Reasoning Lines Transformation

G3: The student will solve problems involving symmetry and transformation.

Focus: Investigating and using formulas, distance, midpoint, and slope

Distance Formula versus **Pythagorean Theorem**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$a^2 + b^2 = c^2$

Given E (-7, -2) and F (5, 3), find the distance

Label Points $x_1,\ y_1$ $x_2,\ y_2$

Substitute into formula

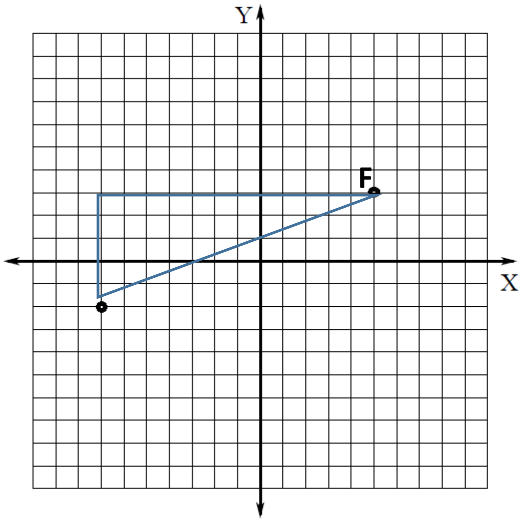
$d = \sqrt{(5-(-7))^2 + (3-(-2))^2}$

$d = \sqrt{(12)^2 + (5)^2}$

$d = \sqrt{144 + 25}$

$d = \sqrt{169}$

$d = 13\ units$



Create a right triangle

Count the units

Use formula

$a^2 + b^2 = c^2$

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$\sqrt{169} = c$

$13\ units = c$

Midpoint Formula

$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

- Middle
- Center
- Half-way
- Bisects – separates into 2 equal parts

Find the midpoint of $\overline{GH}$given $G\ (7, -5)$ and $H\ (9, -1)$

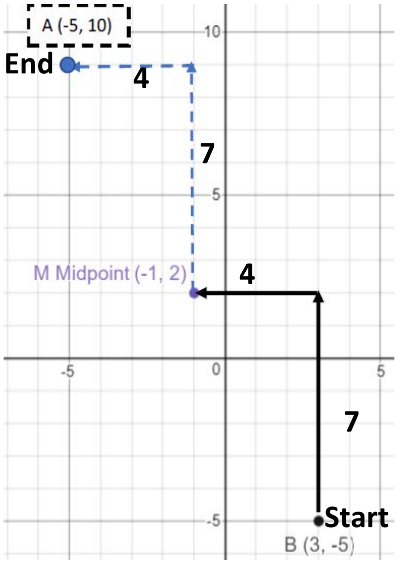
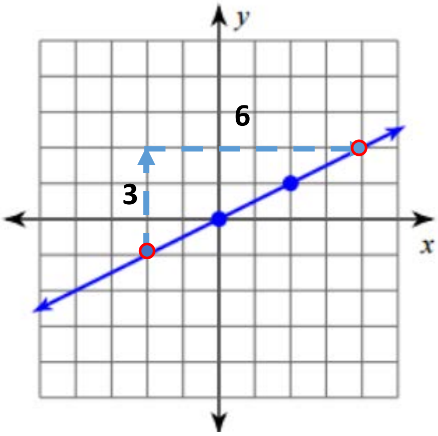
Label your points: $x_1,\ y_1$ $x_2,\ y_2$

Substitute into formula and simplify

$$\frac{7+9}{2}, \frac{-5+(-1)}{2}$$

$$\frac{16}{2}, \frac{-6}{2}$$

Midpoint $(8, -3)$

Finding the Endpoint • Plot the Midpoint and one Endpoint • Use Rise/Run to go from Endpoint to Midpoint. • Repeat the Pattern to find the other Endpoint.		Find the coordinates of A if M (-1, 2) is the midpoint of $\overline{AB}$ and B has coordinates of (3, 5). $\frac{Rise}{Run} = \frac{Up\ 7}{Left\ 4}$
Slope • Slope Formula o $m = \frac{y_2 - y_1}{x_2 - x_1}$ (2 points) • Slope from graph o $\frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$ • Slope from equation: Write in slope intercept form • $y = mx + b$ o $m = slope$ o $b = y - intercept$		**Find the slope** 2 points (-2, -1) and (4, 2) $x_1,\ x_2 \qquad y_1,\ y_2$ $m = \frac{2-(-1)}{4-(-2)} = \frac{3}{6} = \frac{1}{2}$ **Graph** $\frac{Rise}{Run} = \frac{3}{6} = \frac{1}{2}$ **Equation:** $5x + 10y = 21$ $\underline{-5x \qquad\qquad -5x}$ $\frac{10y}{10} = \frac{-5x}{10} + \frac{21}{10}$ $y = \boxed{-\frac{1}{2}}x + \frac{21}{10}$ $m = \frac{1}{2}$

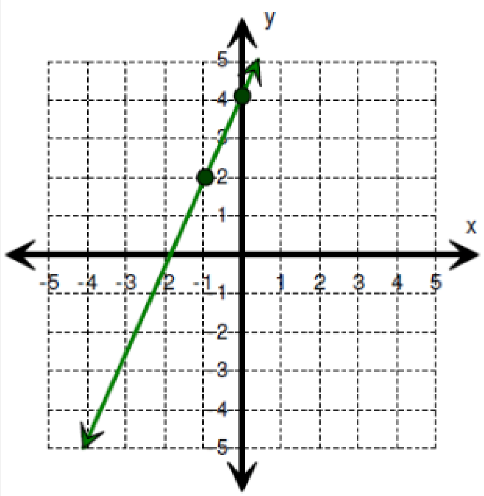
Let’s Practice

1. What is the slope of the equation?

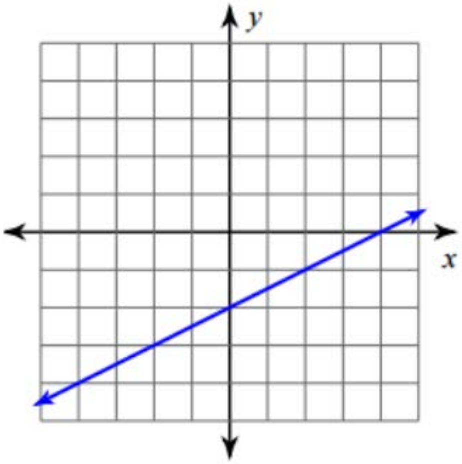
$$4x - 8y = 72$$

2. What is the slope of the line that passes through the points (3, 5) and (-2, 0)?

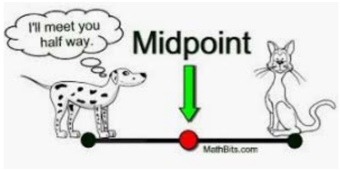
3. Find the slope.



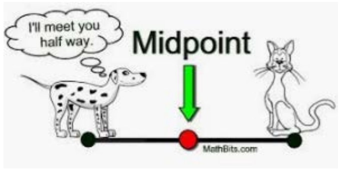
4. Find the slope.



5. Find the midpoint of the segment with the endpoints (6, 7) and (6, -5).

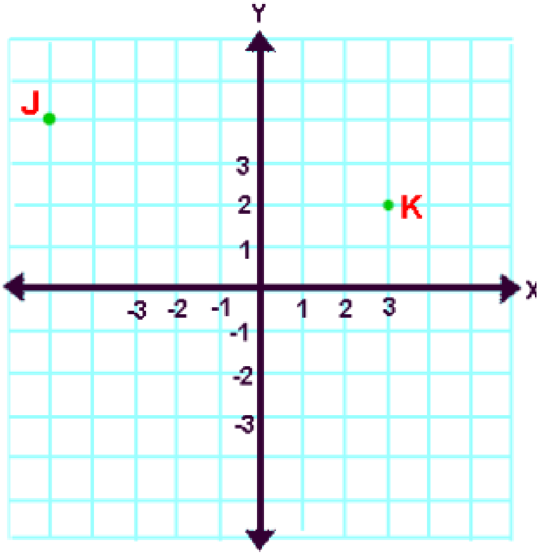


6. Find the midpoint of the segments with the endpoints (1, 7) and (9, 0).

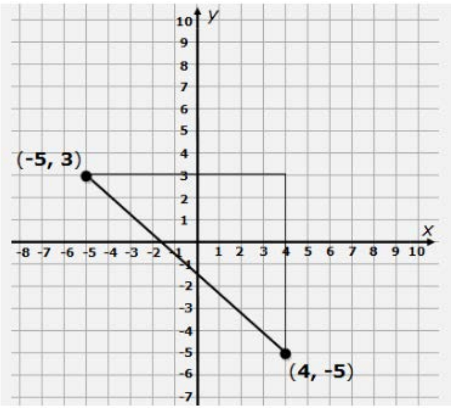


7. What is the distance between (-3, -11) and (-8, -42)?

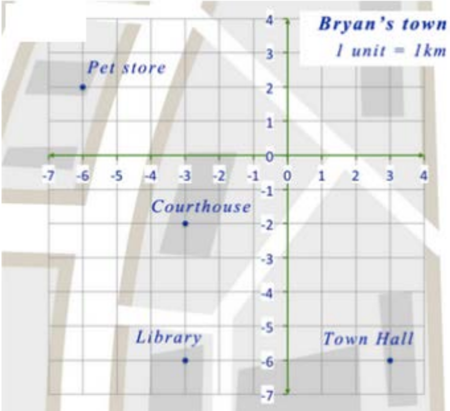
8. Find the distance between points K and J.



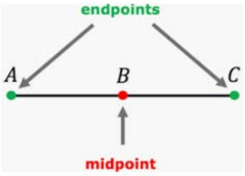
9. What is the distance between (-5, 3) and (4, 5)?



10. Find the distance between the court house and town hall.

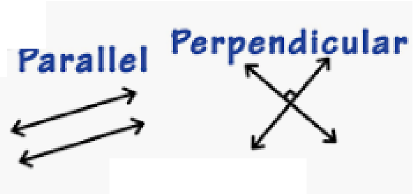


11. Find the missing endpoint if one endpoint is (5, -7) and the midpoint is at (2, 1).



12. Given the endpoint A (-9, -1) and the midpoint at M (-2, -6), determine the other endpoint B.

Focus: Applying slope to verify whether lines are parallel or perpendicular.

Parallel Lines	Have the same slope 
Perpendicular Lines	Have opposite reciprocal (flip) slopes
Opposite Reciprocal (flip) Slopes Examples	$-\frac{8}{7}$ becomes $\frac{7}{8}$ $\frac{3}{4}$ becomes $-\frac{4}{3}$ 6 becomes $-\frac{1}{6}$

Example 1: Use the slope to determine if $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither.

A (-2, 3), B (2, 6), C (-1, 0), and D (3, 3).

Find the slope of $\overleftrightarrow{AB}$. Find the slope of $\overleftrightarrow{CD}$.

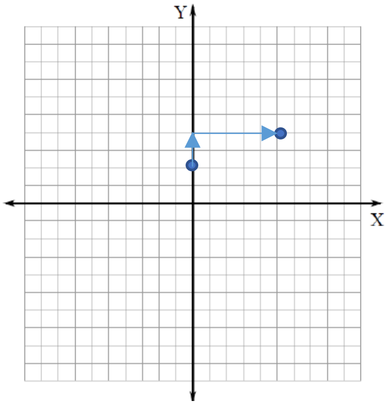
$\frac{6-3}{2-(-2)} = \frac{3}{4}$ $\frac{3-0}{3-(-1)} = \frac{3}{4}$

Compare the slopes:
Since the slope of $\overleftrightarrow{AB}$ equals the slope of $\overleftrightarrow{CD}$, the lines are parallel.

Example 2: Use the slope to determine if $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither.

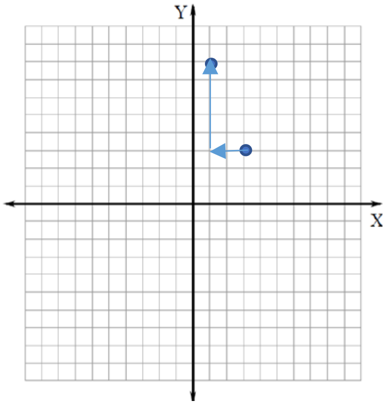
A (0,2), B (5, 4), C (1, 8), and D (3, 3).

Find the slope of $\overline{AB}$



$\frac{rise}{run} = \frac{2}{5}$

Find the slope of $\overline{CD}$



$\frac{rise}{run} = \frac{-2}{5}$

Compare the slopes:
Since the slope of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, neither the same or opposite reciprocal, the slopes are not parallel or perpendicular.

Let’s Practice

1. Slopes of parallel lines are

   ____

2. Slopes of perpendicular lines are

   ____

3. These lines are parallel, perpendicular or neither?

$$\begin{cases} y = 3x + 2 \\ y = 3x - 3 \end{cases}$$

4. These lines are parallel, perpendicular or neither?

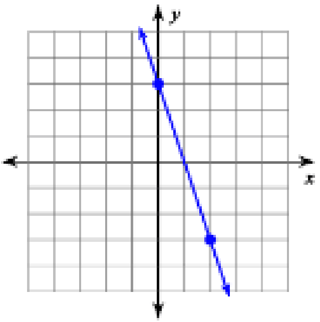
$$\begin{cases} y = -\frac{2}{3}x + 2 \\ y = \frac{3}{2}x + 9 \end{cases}$$

5. What is the slope of the line parallel a line with a slope of $\frac{6}{11}$?

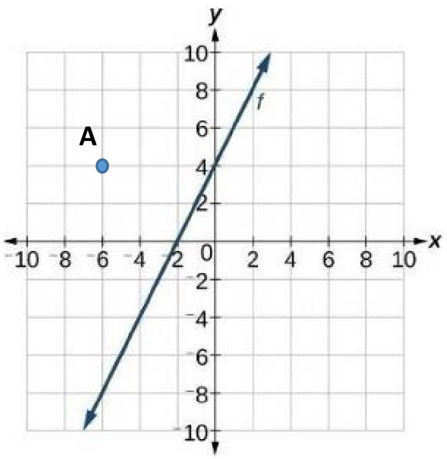
6. Find the slope of the line that is parallel to the line containing the following 2 points.
   (-1, -2) and (3, 3)

7. Find the slope of the line that is perpendicular to the line containing the following two points. (-1, -2) and (3, 3).

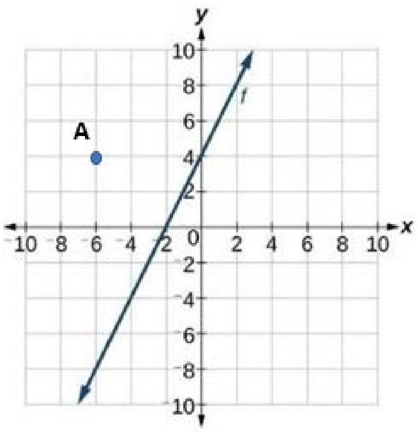
8. What is the slope of the line parallel to the line drawn?

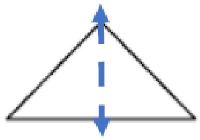
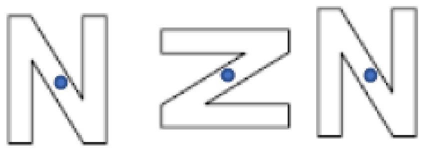


9. Plot one additional point B, so that $\overleftrightarrow{AB}$ is **parallel** to line *f*.



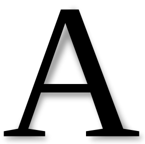
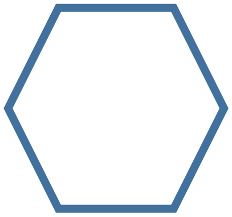
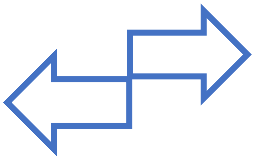
10. Plot one addition point B, so that $\overleftrightarrow{AB}$ is **perpendicular** to line *f*.



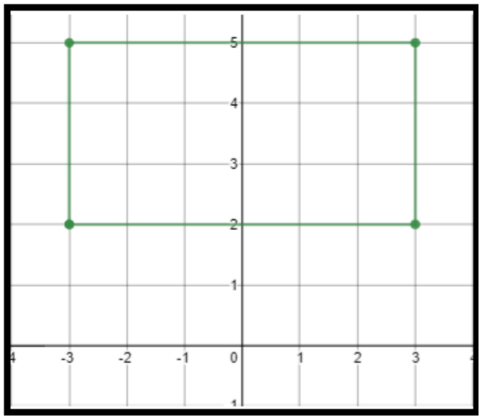
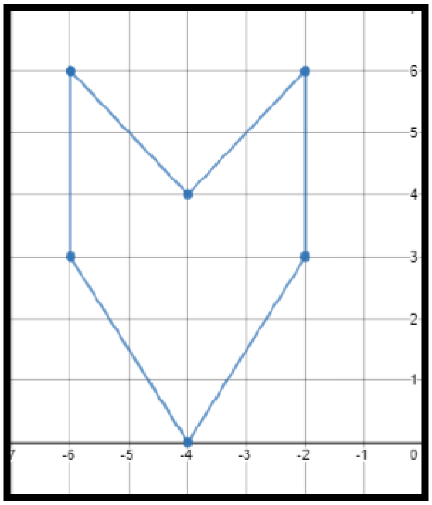
Focus: Investigating symmetry and determine whether a figure is symmetric with respect to a line or a point on a line		
Line Symmetry	A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. (Mirror Image)	
Point Symmetry	A figure has point symmetry if the figure is mapped onto itself by rotating the figure 180° about a center point. The figure will look the same upside down.	 **original** **90°** **180°**

Let’s Practice

Determine if the figures have line and/or point symmetry. Select all correct answers.

1. ☐ Line ☐ Point ☐ None 	2. ☐ Line ☐ Point ☐ None 	3. ☐ Line ☐ Point ☐ None 

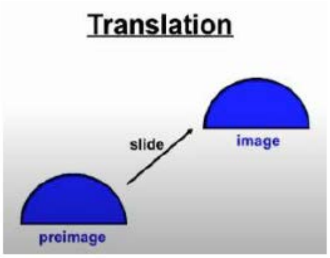
Directions: Find the line of symmetry and write the equation of the line of symmetry.

4. 	5. 

Focus: Determining whether a figure has been translated, rotated, or dilated, using coordinate methods.

I

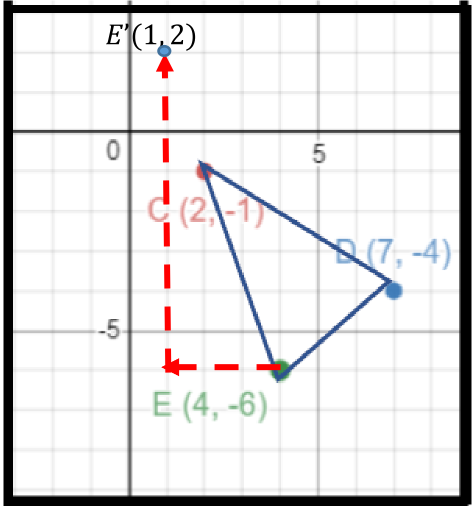
Translation



- To vertically and/or horizontally side a figure.
- Symbolic Form: $(x, y) \to (x + a, y + b)$
- a represents the shift in x (right or left)
- b represents the shift in y (up or down)

Example: Graph and label E'using the rule: $(x, y) \to (x - 3, y + 8)$

x will shift left 3 and y will shift up 8



The coordinates of E' is $(1, 2)$

Algebraically:

$E(4, -6)$

Rule $(x,\ \ y) \to (x - 3, \quad y + 8)$

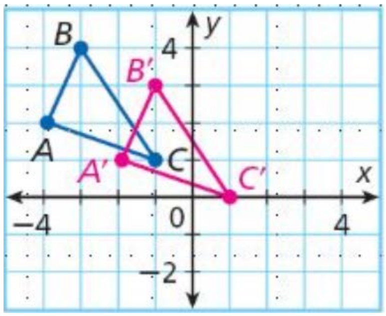
$E\ (4, -6) \to (4 - 3), (-6 + 8)$

$E'(1,2)$

Let's Practice

1. Triangle ABC is translated by the rule $(x, y) \to (x + 3, y - 2)$. What will be the position of A'?

2. Write a rule for the translation.

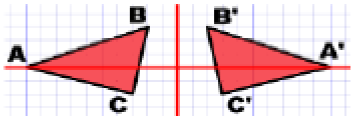


3. Given Trapezoid STUV with T (2, -5), using the rule $(x, y) \to (x + 3, y - 6)$ find T'.

4. If $B\ (2, 4)$ is translated resulting in $B'\ (4, -2)$, how was B translated?

Reflection

A flip (mirror image) over al line called the Line of Reflection.



Each point in its image are the same distance from the line of reflections.

Note

In addition, you can have a vertical reflection $x = c$ or a horizontal reflection $y = c$ where "c" is a constant.

Possible lines of reflections and rules.

Reflection	Rule
Across the x-axis	$(x, y) \to (y, -x)$
Across the y-axis	$(x, y) \to (-x, y)$
Across the line y=x (diagonal line)	$(x, y) \to (y, x)$
Across the line y = -x (diagonal line)	$(x, y) \to (-y, -x)$

Example 1: Triange ABC with vertices: A (-4, 2), B (4, 7) and C (5, 1) is reflected across the x-axis, what are the coordinates B' of the new image.

Rule: $(x, y) \to (y, -x)$

$B\ (4, 7) \to B'(7, -4)$

Example 2: Square ABCD with vertices $A\ (-1, 3), B\ (0,6), C\ (3, 5)$, and $D\ (2\ , 3)$ is reflected across the line $y = -x$. What are the coordinate of A'?

Rule: $(x, y) \to (-y, -x)$

$A\ (-1, 3) \to A'(-3, 1)$

Let's Practice

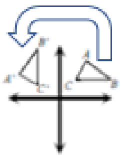
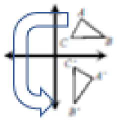
1. Triangle FGH with vertices $F(1, 8), G\ (5, 7)$, and $H\ (2, 3)$ is reflected across the line $y = x$, Find H'.

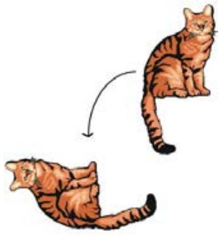
2. Rectangle PQRS with vertices: $P(1, 2), Q(2, 5), R\ (8, 3)$ and $S\ (7, 0)$ is reflected across the y-axis, what is Q'?

3. Parallelogram $F\ (4, -3)$ is one of the vertices of Parallelogram CDEF. What is F' if the figure is reflected across the x-axis.

4. Triangle JKL with vertices $J\ (1, -1), K\ (2, 3)$, and $L\ (3, -2)$ is reflected across $y = -x$. What are the coordinates of L'?

Rotation

Type of Rotation	Pictorial	Rule
Rotation 90° Clockwise		$(x,y) \rightarrow (y,-x)$
Rotation 90° Counterclockwise		$(x,y) \rightarrow (-y,x)$
Rotation 180° clockwise and counterclockwise		$(x,y) \rightarrow (-x,-y)$
Rotation 270° Clockwise		$(x,y) \rightarrow (-y\,,x)$
Rotation 270° Counterclockwise		$(x,y) \rightarrow (y,-x)$



A turn around a fixed point is called the center of rotation. The figure rotates at a specific angle and direction.

Example 1: ΔABC with vertices $A\ (2,7), B\ (6,5)$ and $C\ (4,1)$ is rotated 90° counterclockwise. Find C'.

Rule: $(x,y) \rightarrow (-y,x)$

$C\ (4,1) \rightarrow C'(-1,4)$

Example 2: Triangle LMN with vertices $L\ (1,8), M\ (5,7)$ and N $(2,3)$ is rotated 270° counterclockwise. What are the coordinates of M'?

Rule: $(x,y) \rightarrow (y,-x)$

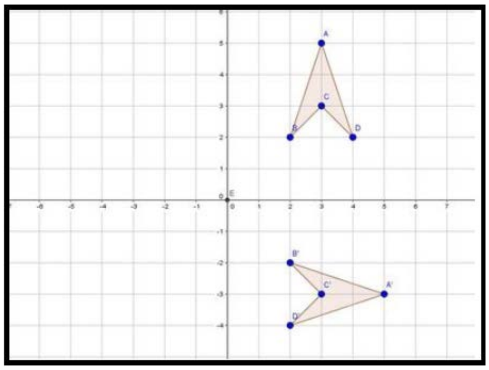
$M(5,7) \rightarrow M'(7,-5)$

Let's Practice

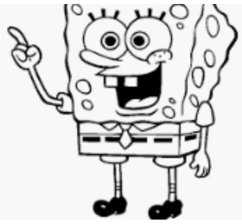
1. If you were to rotate ABCD 180° about the origin what would be the coordinates of B' given $B\ (5,-2)$?

2. The point $P\ (7,8)$ is rotated 90 degrees counterclockwise. What is the P'?

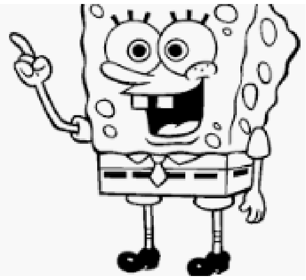
3. What is the degree of rotation for this counterclockwise rotation about the origin?

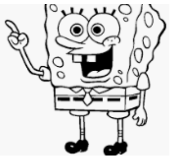


Dilations



Original





reduction

Enlargement

The enlargement or reduction of a figure. The scale factor indicates how much the figure will enlarge or reduce.

"K" is a variable for scale factor.

- When $K > 1$, then the dilation is an enlargement.
    - Rule multiply by K
- When $0 < K < 1$, then the dilation is a reduction.
    - Rule multiply by K

Example 1: Triangle RST with vertices $R\ (-5,1), S\ (-3,4), and\ T\ (2,-1)$ is dilated using $K = 2$. Find $R', S' and\ T'$.

Rule: $(x,y) \to (xK, yK)$

$R\ (-5,1) \to R'(-5 \cdot 2, 1 \cdot 2)$

$R'\ (-10,2)$

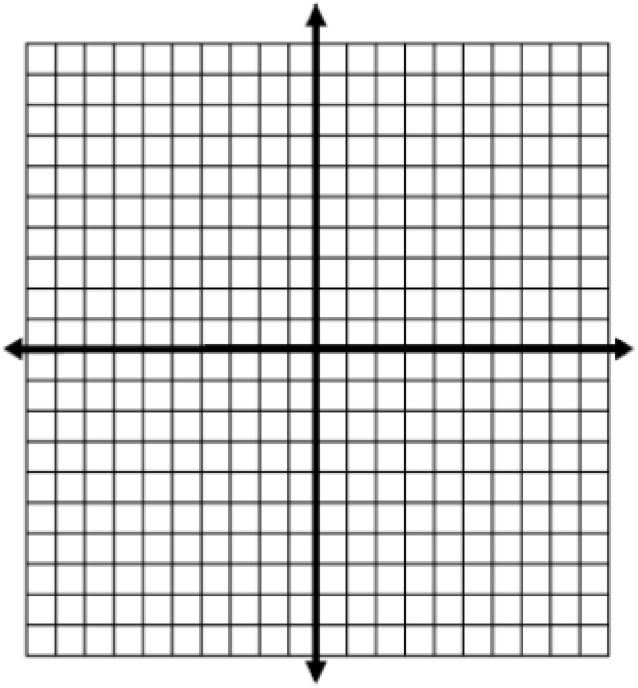
Example 2: Rhombus JKLM with vertices $J\ (-10,2), K\ (2,8), L\ (6,2)$ and $M\ (-2,-4)$ is dilated with a scale factor of $\frac{1}{2}$ find K'.

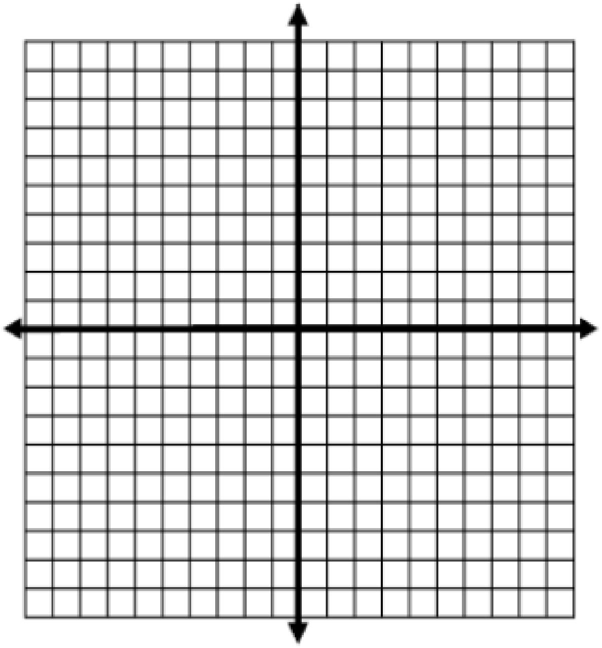
Rule: $(x,y) \to (xK, yK)$

$K\ (2,8) \to \left(2 \cdot \frac{1}{2}, 8 \cdot \frac{1}{2}\right)$

$K'(1,4)$

Let's Practice – Using Examples 1 and 2 above plot the image and pre-image below.



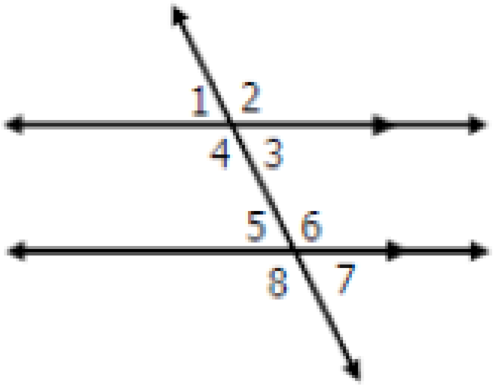


Week 2

G.2: The student will use the relationships between angles formed by two lines intersected by a transversal to
a) prove two or more lines are parallel; and
b) solve problems, including practical problems, involving angles formed when parallel lines are intersected by a transversal.

Notes:

- Parallel lines intersected by a transversal form angles with specific relationships.



- Some angle relationships may be used when proving two lines intersected by a transversal are parallel.
- **To prove that 2 lines are parallel, you must be able to prove:**

✓ corresponding angles are congruent OR
✓ alternate interior angles are congruent OR
✓ alternate exterior angles are congruent OR
✓ consecutive interior angles are supplementary OR
✓ both lines are perpendicular to the same line OR
✓ both lines are parallel to the same line OR
✓ the two lines have the same slope.

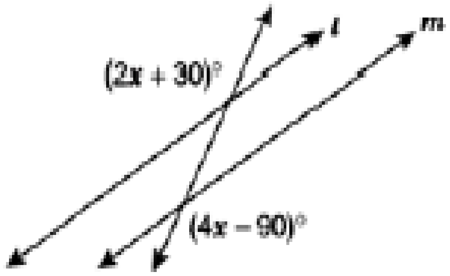
Slope = $\frac{y_2 - y_1}{x_2 - x_1}$

Parallel lines – have the same identical slope

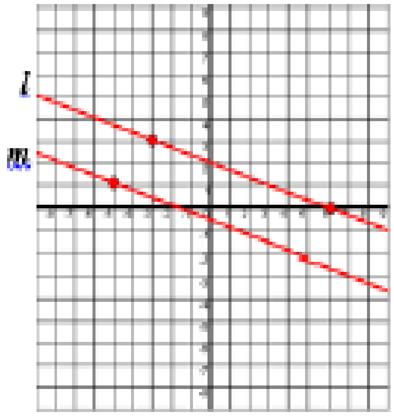
Perpendicular lines – have negative reciprocal slopes

Day 1

1) What value for x will show that lines l and m are parallel?



2) Prove that line l is parallel to line m using slope.

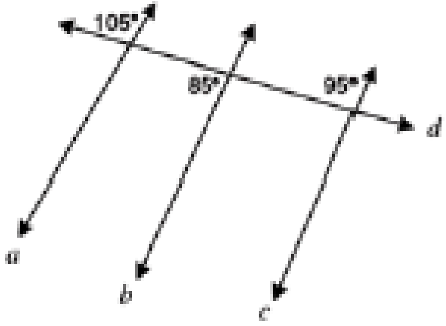


Slope of Line l:

Slope of Line m:

Why are they parallel?

3) In this diagram, line d cuts three lines to form the angles shown.



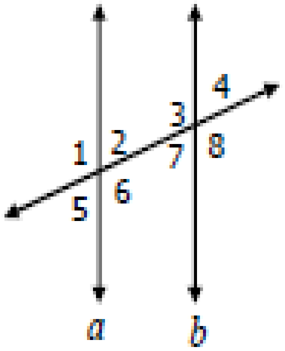
Which two lines are parallel?

A a and b
B a and c
C b and c
D No lines are parallel

4) Which statement describes the lines whose equations are $y = \frac{1}{3}x + 12$ and $6y = 2x + 6$?

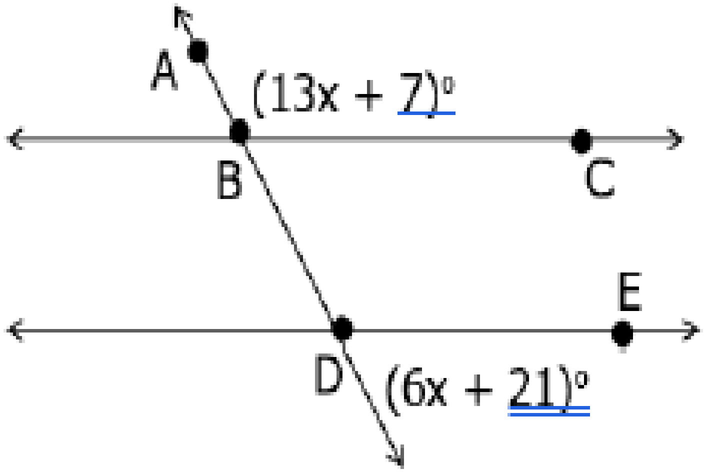
A They are segments
B They are perpendicular to each other
C They intersect
D They are parallel to each other

5) Given: ∠1 and ∠2 form a linear pair; ∠1 and ∠4 are supplementary
Prove: $a \,||\, b$

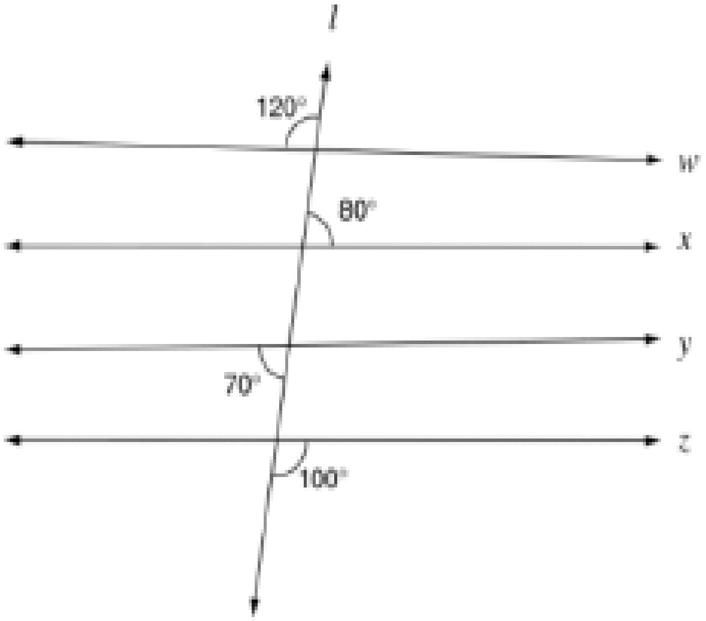


Stations	Reasons
1. ∠1 and ∠2 form a linear pair	1.
2. ∠1 and ∠2 are supplementary	2.
3. ∠1 and ∠4 are supplementary	3.
4. ∠2 ≅ ∠4	4.
5. $a \,\|\|\, b$	5.

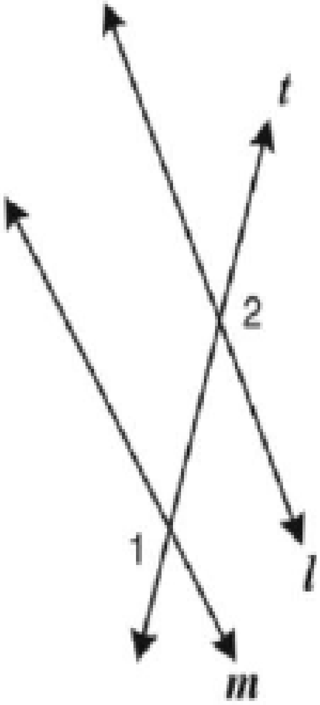
6) What measure of ∠ABC will prove that $\overleftrightarrow{BC}$ is parallel to $\overleftrightarrow{DE}$?



7) Line *l* intersects lines w, x, y, and z. Which two lines are parallel?



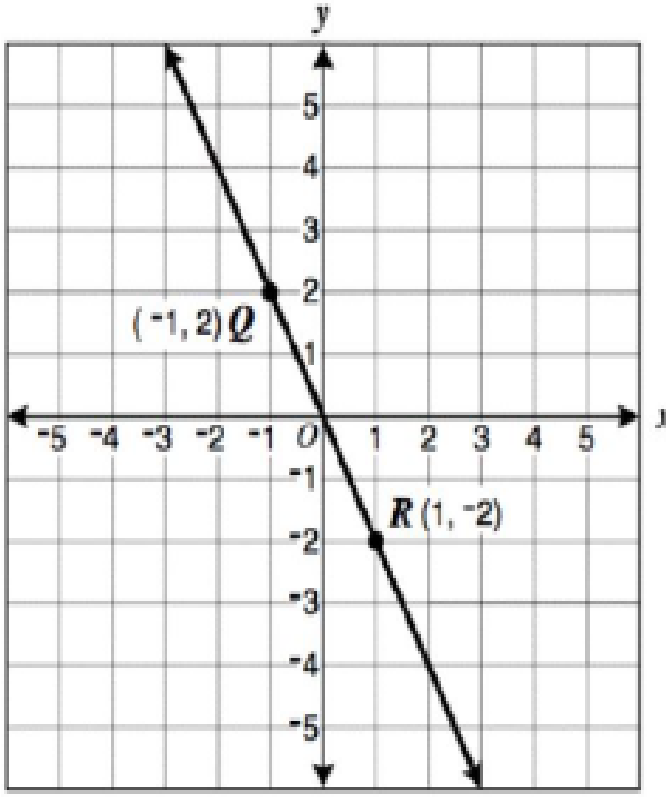
8) In the figure, line *t* is a transversal for lines *l* and *m*.



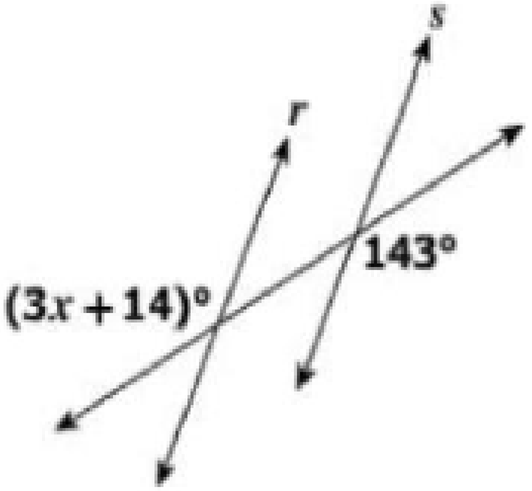
Which of the following best describes the relationship between ∠1∠ and ∠2?

A) Consecutive interior angles.
B) Alternate exterior angles
C) Corresponding angles
D) Alternate interior angles.

9) Find any two points that will determine a line parallel to line QR?

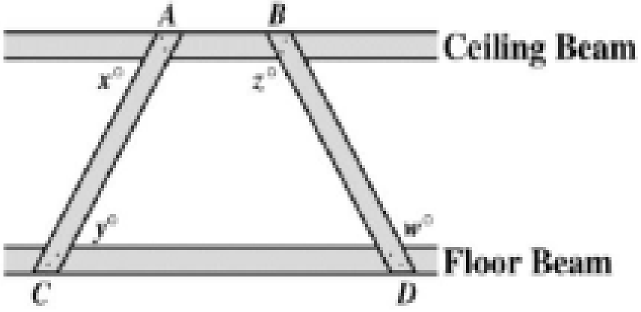


10) Lines *r* and *s* are cut by a transversal. What value of *x* proves that $r \parallel s$?

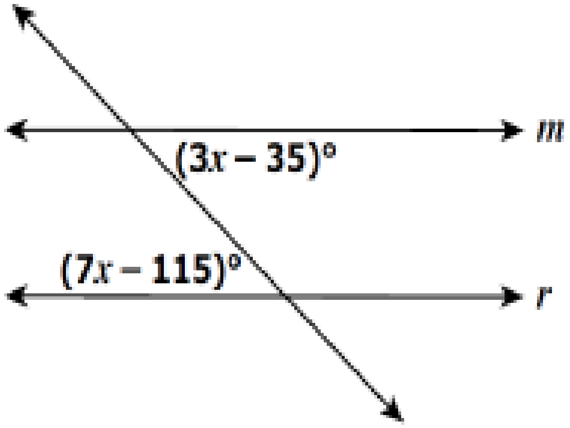


Day 2

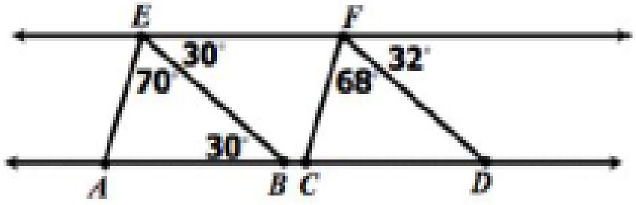
1) A construction engineer needs to make sure a ceiling beam is parallel to its corresponding floor beam. Using the drawing as a guide, which pair of measurements is sufficient to show the beams are parallel?



2) Lines *m* and *r* are cut by a transversal. What value of *x* will show that line *m* is parallel to line *r*?



3) Determine which statement best represents the information in the figure.



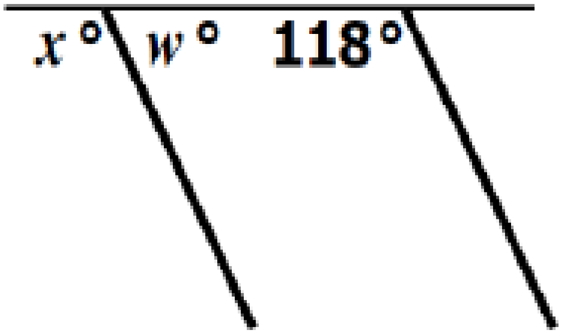
4) This figure represents line segments painted on a parking lot to create parking spaces.

$\overline{AE} \parallel \overline{CF}$ and $\overline{EF} \parallel \overline{AD}$

$\overline{AD} \parallel \overline{EF}$ and $\overline{CF} \parallel \overline{FD}$

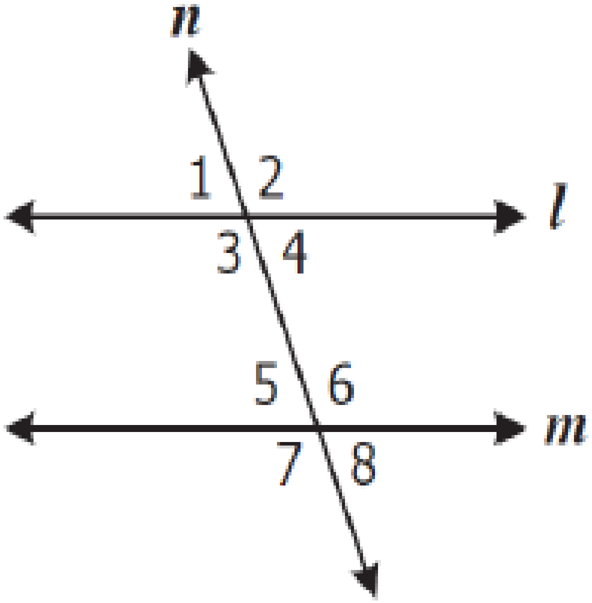
$\overline{AD} \parallel \overline{EF}$ and $\overline{EB} \parallel \overline{FD}$

$\overline{AE} \parallel \overline{CF}$ and $\overline{EB} \parallel \overline{FD}$

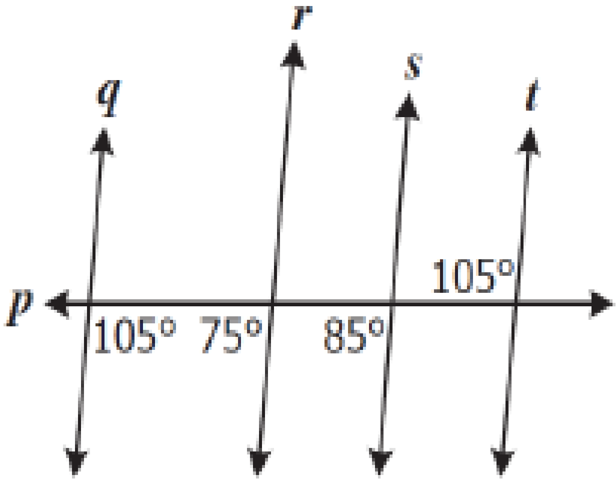


Write an that equation can be used to show that these line segments are parallel?

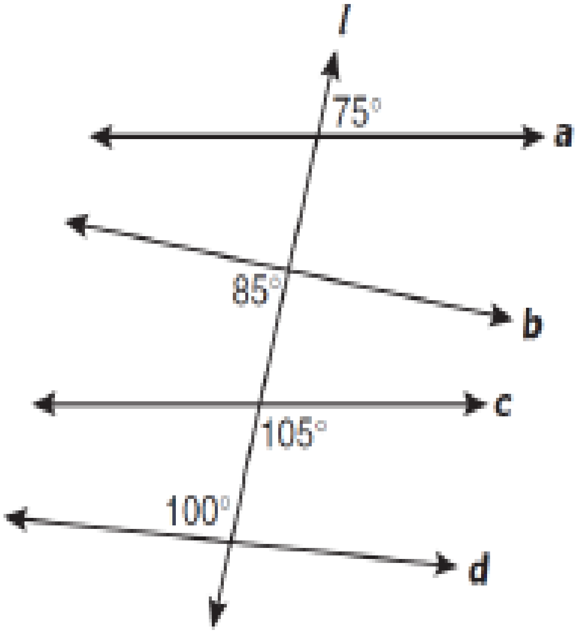
5) Lines *l* and *m* are cut by transversal *n*. Which angle pairs or angle pair relationships would prove that lines *l* and *m* are parallel?



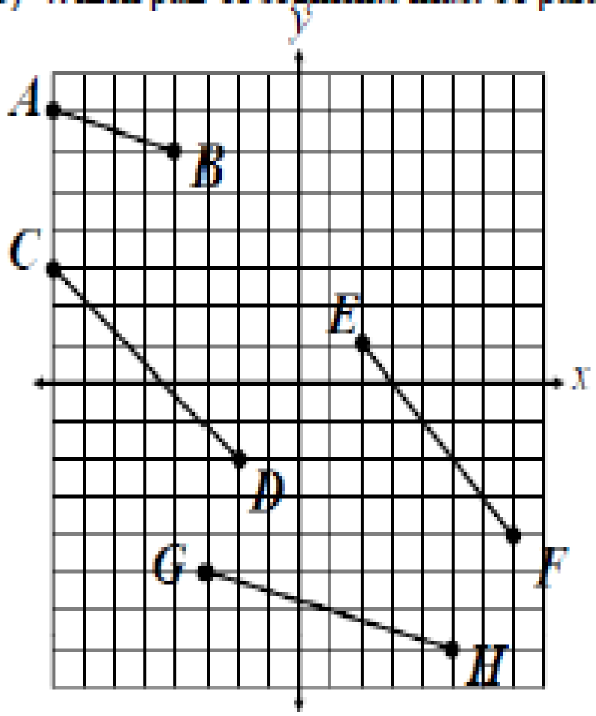
6) Line *p* is a transversal. For lines *q*, *r*, *s*, and *t*, which is not parallel to the other three?



7) Transversal l cuts lines a, b, c, and d. Which two lines are parallel?



8) Which pair of segments must be parallel?



Explain how you determined your answer.

9) Which line is parallel to the line $y = x + 7$?

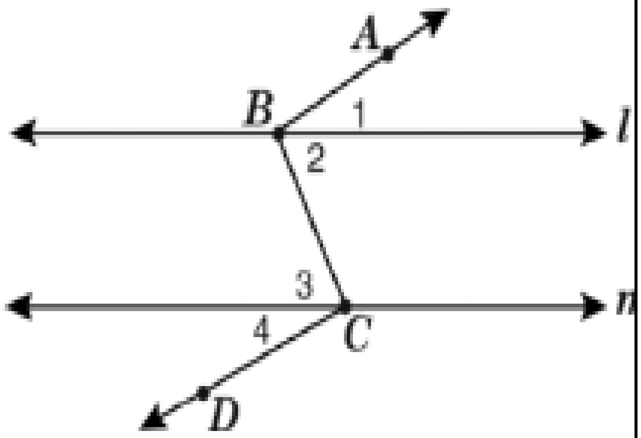
A) $x - y = 5$

B) $x + y = -2$

C) $y = 1$

D) $x = 1$

10) Use this figure to answer the following.



$\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$ if

A) $m\angle 3 = m\angle 4$

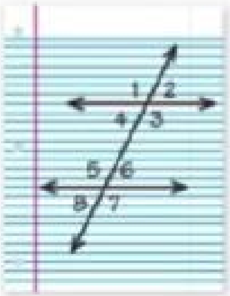
B) $m\angle 1 = m\angle 2$

C) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$

D) $m\angle 1 + m\angle 2 = 90$

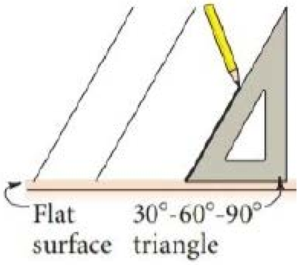
PROJECT Draw two horizontal lines and a transversal on a piece of notebook paper. Label the angles as shown. Use a pair of scissors to cut out the angles. Compare the angles to determine which angles are congruent.

REASONING Refer to the figure for Exercise 13. What is the least number of angle measures you need to know in order to find the measure of every angle? Explain your reasoning.



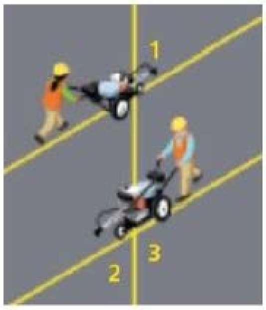
Crew: If the rowing crew strokes in unison, the oars sweep out angles of equal measure. Explain why the oars on each side of the shell stay parallel.

Drafting An artist uses the drawing tool in the diagram at the right. The artist draws a line, slides the triangle along the flat surface, and draws another line. Explain why the drawn lines must be parallel.



Real-World Connection

Parking Two workers are painting lines for angled parking spaces. The first worker paints a line so that $m\angle 1 = 60^\circ$. The second worker paints a line so that $m\angle 2 = 60^\circ$. Explain why their lines are parallel.



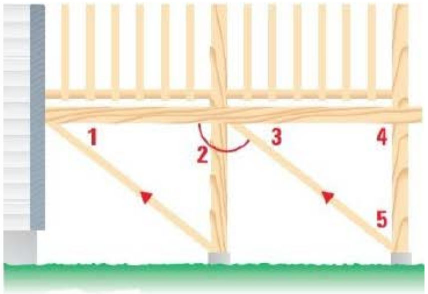
If the second worker uses $\angle 3$, what should $m\angle 3$ be for parallel lines? Explain.

CONSTRUCTION Two posts support a raised deck. The posts have two parallel braces, as shown.

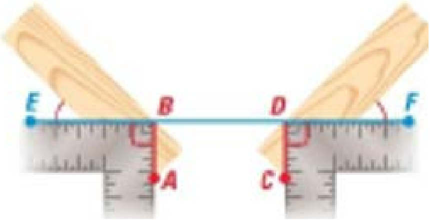
a. If $m\angle 1 = 35^\circ$, find $m\angle 2$.

b. If $m\angle 3 = 40^\circ$, what other angle has a measure of 40°?

c. Each post is perpendicular to the deck. Explain how this can be used to show that the posts are parallel to each other.



A carpenter wants to cut two boards to fit snugly together. The carpenter's squares are aligned along $\overline{EF}$, as shown. Are $\overline{AB}$ and $\overline{CD}$ parallel? State the theorem that justifies your answer.

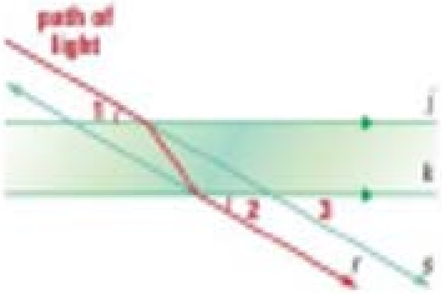


Fireplace Chimney: In the illustration at the right ∠ABC and ∠DEF are supplementary. Explain how you know that the left and right angles of the chimney are parallel.

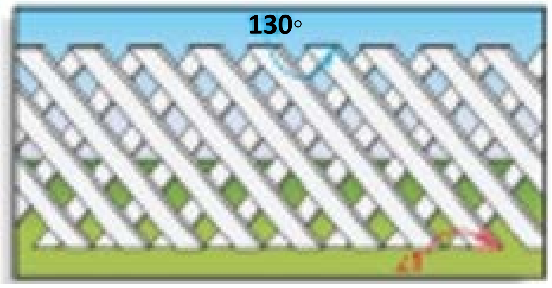
Science Connection: When light enters glass, the light bends. When it leaves glass, it bends again. If both sides of a pane of glass are parallel, light leaves the pane at the same angle at which it entered. Prove that the path exiting light is parallel to the path of the emerging light.

GIVEN: ∠1 ≅ ∠2; j ∥ k

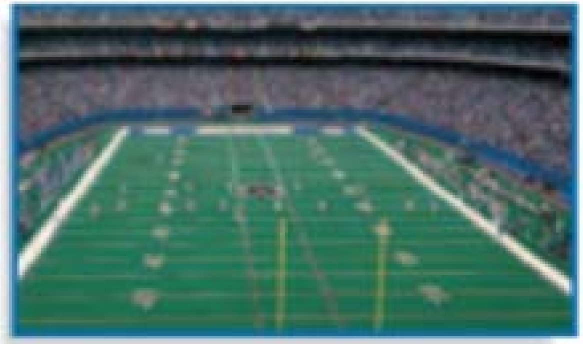
PROVE: r ∥ s



Latticework: You are making a lattice fence out of pieces of wood called slats. You want the top of each slat to be parallel to the bottom. At what angle should you cut ∠**1**?

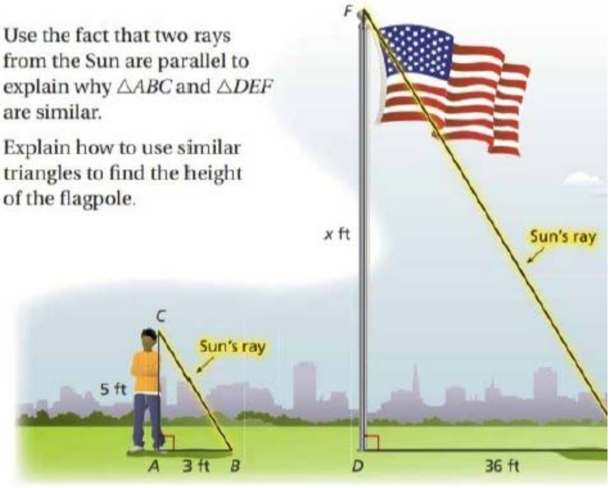


Football Field: The white lines along the long edges of a football field are called sidelines. Yard lines are perpendicular to the sidelines and cross the field every five yards. Explain why you can conclude that the yard lines are parallel.



a. Use the fact that two rays from the Sun are parallel to explain why $\triangle ABC$ and $\triangle DEF$ are similar.

b. Explain how to use similar triangles to find the height of the flagpole.



In Excersie 3-6, use the figure.
3. Identify the parallel lines

4. Identify the transversal

5. How many angles are formed by the transversal?

6. Which of the angles are congruent?

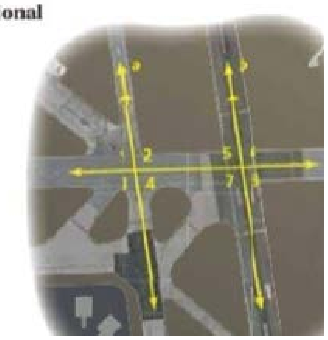
A store uses pieces of tape to paint the window advertisement. The letters are slanted at an 80◦ angle. What is the measure of ∠1?

80◦ B. 100◦ C. 110◦ D. 120◦

The photo shows a portion of the St. Petersburg-Clearwater International Airport. Describe the relationship between each pair of angles.

a. ∠3 and ∠6

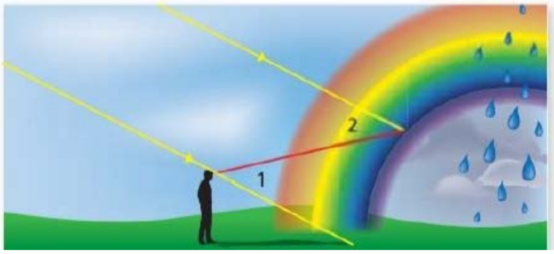
b. ∠2 and ∠7



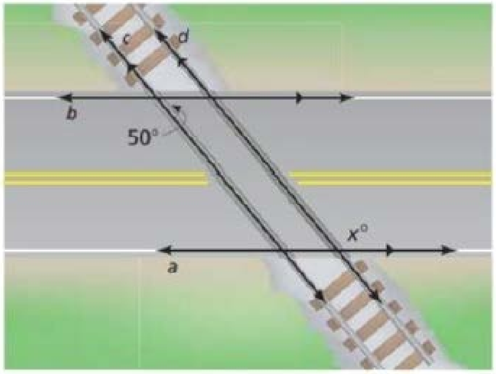
A.

PARKING The painted lines that separate parking spaces are parallel. The measure of ∠1 is 60°. What is the measure of ∠2? Explain.

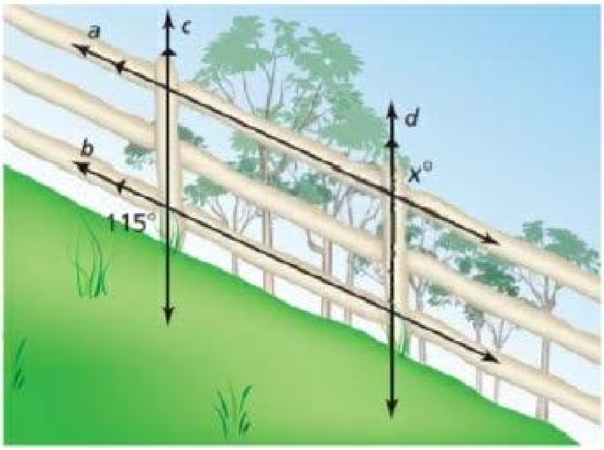
Rainbow: A rainbow is formed when sungligh reflects off raindrops at different angles. For blue light, the measure of ∠2 is 40◦. What is the measure of ∠1?

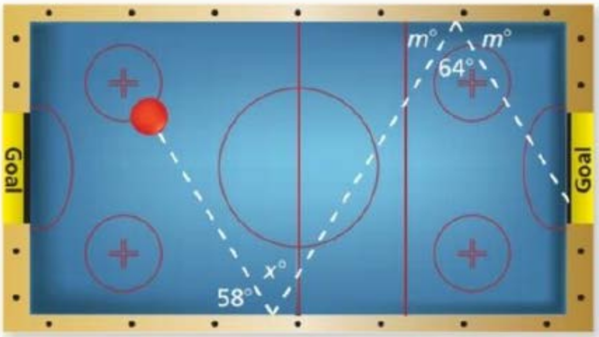


CRITICAL THINKING Find the value of x.



Find the value of x.





Geometry The figure shows the angles used to make a double bank shot in an air hockey game.

a. Find the value of x.

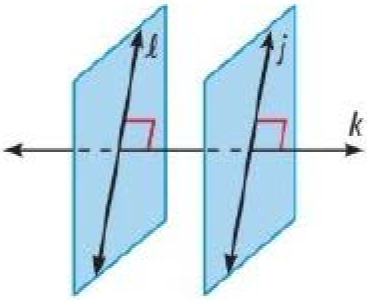
b. Can you still get the red puck in the goal if x is increased by a little? by a lot? Explain.

Carpentry: A T-bevel is a tool used by carpenters to draw congruent angles. By loosening the locking lever, the carpenter can adjust the angle. Explain how the carpenter knows that two lines drawn unsing the T-bevel are prallel.

Real-World Connection

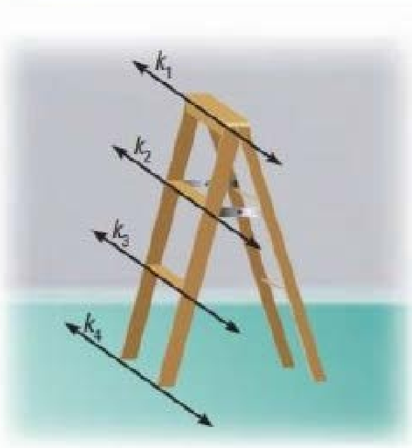
Careers A carpenter must draw angles precisely to ensure good fit.

ERROR ANALYSIS It is given that $j \perp k$ and $k \perp \ell$. A student reasons that lines j and ℓ must be parallel. What is wrong with this reasoning? Sketch a counterexample to support your answer.



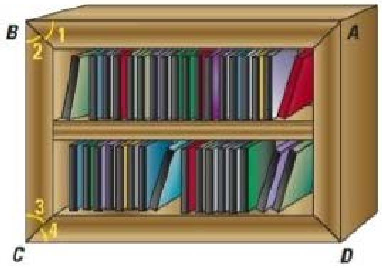
Explaining Why Steps are Parallel

In the diagram at the right, each step is parallel to the step immediately below it and the bottom step is parallel to the floor. Explain why the top step is parallel to the floor.



Building a CD Rack

You are building a CD rack. You cut the sides, bottom, and top so that each corner is composed of two 45° angles. Prove that the top and bottom front edges of the CD rack are parallel.



SOLUTION

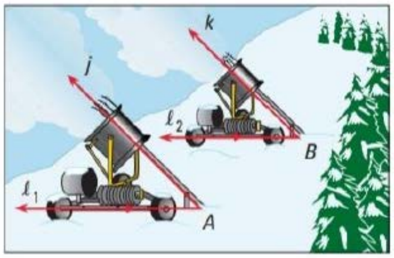
GIVEN ▶ $m\angle 1 = 45°, m\angle 2 = 45°$
$m\angle 3 = 45°, m\angle 4 = 45°$

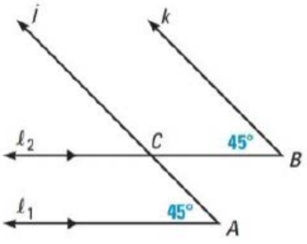
PROVE ▶ $\overline{BA} \parallel \overline{CD}$

SNOW MAKING To shoot the snow as far as possible, each snowmaker below is set at a 45° angle. The axles of the snowmakers are all parallel. It is possible to prove that the barrels of the snowmakers are also parallel, but the proof is difficult in 3 dimensions. To simplify the problem, think of the illustration as a flat image on a piece of paper. The axles and barrels are represented in the diagram on the right. Lines j and ℓ_2 intersect at C.

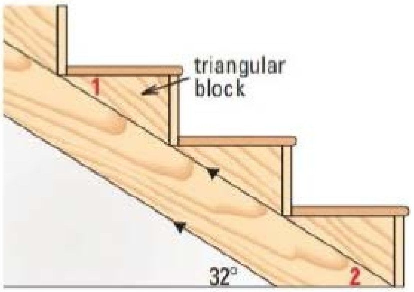
GIVEN ▶ $\ell_1 \parallel \ell_2, m\angle A = m\angle B = 45°$

PROVE ▶ $j \parallel k$





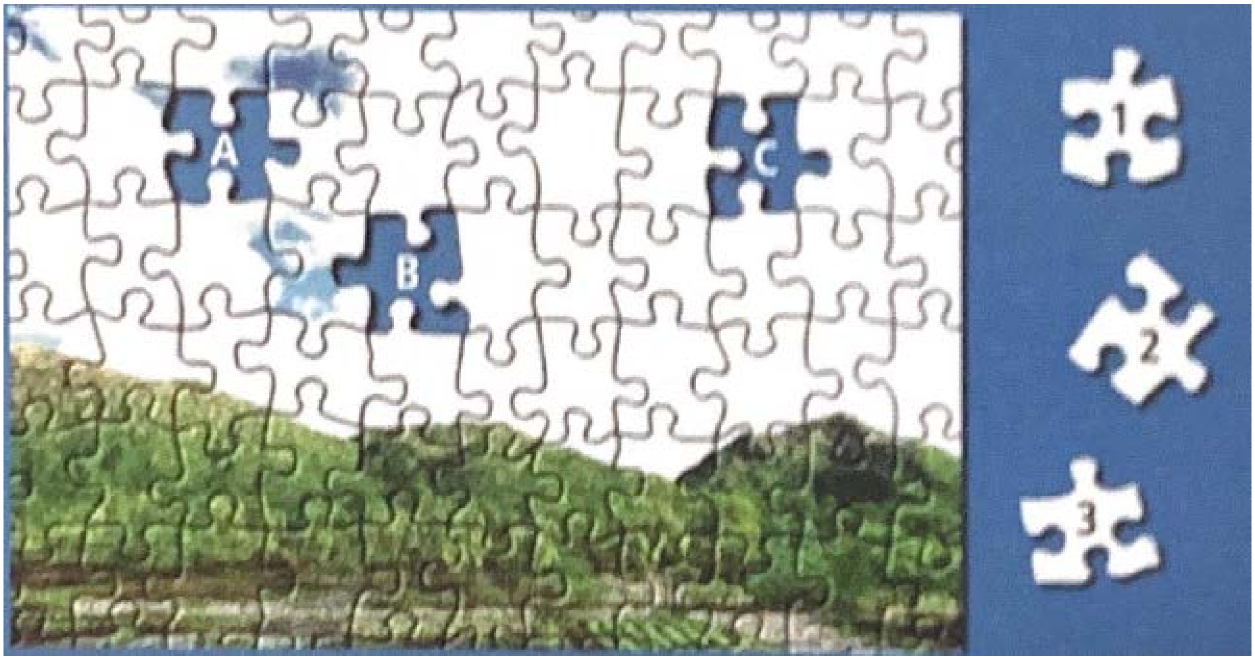
BUILDING STAIRS One way to build stairs is to attach triangular blocks to an angled support, as shown at the right. If the support makes a 32° angle with the floor, what must $m\angle 1$ be so the step will be parallel to the floor? The sides of the angled support are parallel.



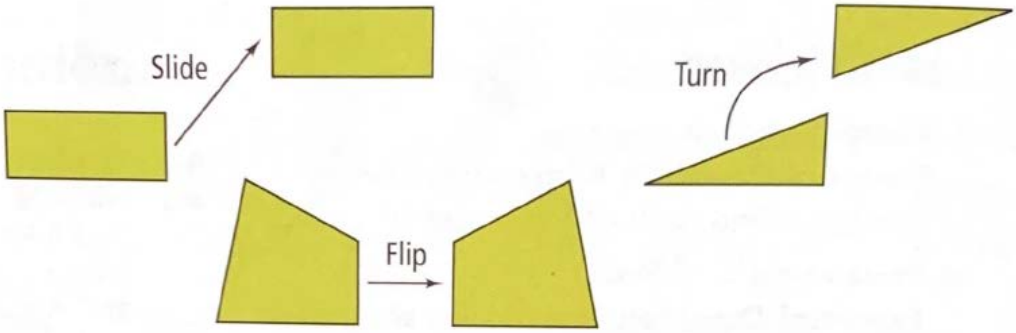
Week 3

PART 1 - Congruent Triangles Notes

You are working on a puzzle. You've almost finished, except for a few pieces of the sky. Imagine putting the remaining pieces in the puzzle. How did you figure out where to place the pieces?



Congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one, as shown below. In this lesson, you will learn how to determine if geometric figures are congruent.



Focus Question How can you recognize congruent figures and their corresponding parts?

Congruent Symbol

≅

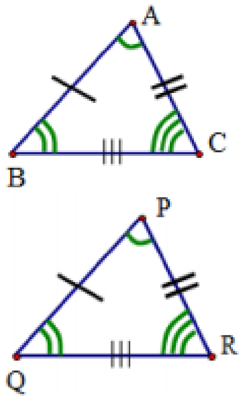
Congruent Triangles –

EXACTLY the same shape and size

Triangle Congruency Statement: Shows the sides and angles that are congruent

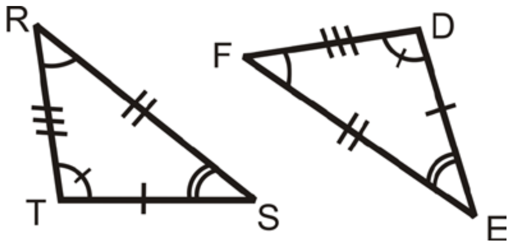
ΔABC ≅ ΔPQR

Hint: Use the ANGLES to write the congruency statement



Sides:	Angles:
AB ≅ PQ	∠A ≅ ∠P
AC ≅ PR	∠B ≅ ∠Q
BC ≅ QR	∠C ≅ ∠R

Remember, to match up corresponding sides and angles, you can do this in different ways.



The "parts" with the same congruence mark mean those parts correspond with each other. In this diagram, you can see that:

∠R ≅ ∠F

∠T ≅ ∠D

∠S ≅ ∠E

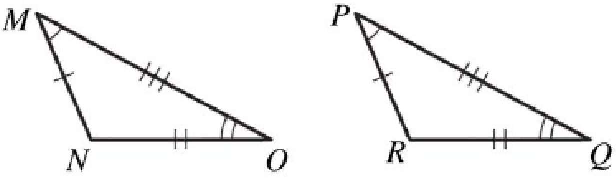
and

$\overline{RS} \cong \overline{FE}$

$\overline{RT} \cong \overline{FD}$

$\overline{TS} \cong \overline{DE}$

Example 2 – with a congruence statement



$\Delta MNO \cong \Delta PRQ$

This time you do not even need to look at the diagram, everything you need to know is all in the congruence statement. Order matters so the position of the lettering tells you everything that is corresponding. The statement tells me that:

∠M ≅ ∠P

∠N ≅ ∠R

∠O ≅ ∠Q

and

$\overline{MN} \cong \overline{PR}$

$\overline{NO} \cong \overline{RQ}$

$\overline{MO} \cong \overline{PQ}$

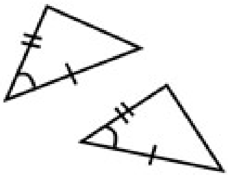
Methods that **Prove Triangles Congruent**

The following ordered combinations of the congruent triangle facts
will be sufficient to prove triangles congruent.

SSS
Side-Side-Side

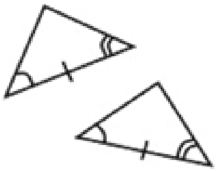
If three sides of a triangle are congruent to three sides of another triangle, the triangles are congruent.

SAS
Side-Angle-Side



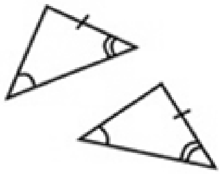
If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.

ASA
Angle-Side-Angle



If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.

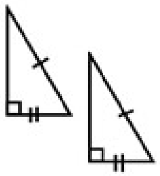
AAS (or **SAA**)
Angle-Angle-Side



If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.

This is an extension of ASA. In ASA, since you know two sets of angles are congruent, you automatically know the third sets are also congruent since there are 180° in each triangle.

HL
Hypotenuse-Leg



If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent.

The "included angle" in SAS is the angle formed by the two sides of the triangle being used.
The "included side" in ASA is the side between the angles being used. It is the side where the rays of the angles overlap.
The "non-included" side in AAS can be either of the two sides that are not directly between the two angles being used.

Once triangles are proven congruent, the corresponding leftover "parts" that were not used in SSS, SAS, ASA, AAS and HL, are also congruent.

CPCTC

Corresponding Parts of Congruent Triangles are Congruent.

Methods that **DO NOT** Prove Triangles Congruent

AAA
Angle-Angle-Angle

SSA or ASS
Side-Side-Angle

Tips for Preparing Congruent Triangle Proofs:

When working with congruent triangles, remember to:

1. Start by marking the given information on your diagram (using hash marks, arcs, etc.).
2. Remember your definitions! If the given information contains definitions, be sure to use them as they are "hints" to the solution.
3. Look for any parts that your triangles may "share". These common parts will automatically be one set of congruent parts.
4. If you are missing needed pieces to prove the triangles congruent, examine the diagram to see what else you may already know about the figure.
5. If you are trying to prove specific "parts" of the triangles are congruent, find a set of triangles that contains these parts and prove those triangles congruent.
6. If the triangles you need are overlapping, try drawing the two triangles separately. It may give you a better look at the known information.
7. Keep in mind that there may be more than one way to solve the problem.

A proof is like a big "puzzle" waiting to be solved. Look carefully at the "puzzle" and use all of your geometrical strategies to arrive at a solution.

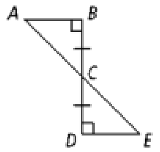
Some of the more common theorems, properties, and definitions used with congruent triangles:

1. Reflexive Property - when a quantity is equal (or congruent) to itself. Used for shared parts.
2. Transitive Property - if two quantities are = to the same quantity, they are = to each other.
3. Angle Bisector - a ray in the interior of an angle creating two congruent angles.
4. Segment Bisector - a line, segment or ray that divides the segment into two congruent parts.
5. Midpoint of Segment - a point on the segment creating two congruent segments.
6. Median of a Triangle - a segment from any vertex of a Δ to the midpoint of the opposite side.
7. Altitude of a Triangle - a segment from any vertex of a Δ perpendicular to the line containing the opposite side.
8. Vertical angles are congruent. These are the angles in the corners of an X.
9. Right angles are congruent.
10. If two angles form a linear pair, they are supplementary.
11. Points that lie on a perpendicular bisector of a segment are equidistant from the ends of the segment.
12. If two parallel lines are cut by a transversal, the alternate interior angles are congruent.

Of course, there are more theorems, properties and definitions that may be used.

Week 3 Part 1: PRACTICE WITH CONGRUENCY

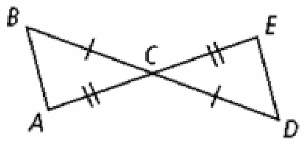
1) Given: $\overline{BD} \perp \overline{AB}$, $\overline{BD} \perp \overline{DE}$,
$\overline{BC} \cong \overline{DC}$
Prove: $\angle A \cong \angle E$



Thoughts:

Statements	Reasons

2) Given: $\overline{BC} \cong \overline{DC}, \overline{AC} \cong \overline{EC}$
Prove: $\Delta ABC \cong \Delta EDC$

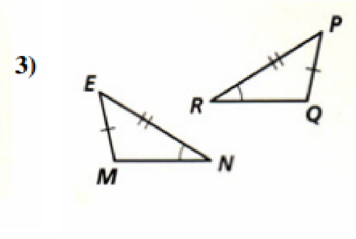


Thoughts:

Statements	Reasons

For the following questions, determine if the triangles are congruent. If they are, state what postulate you used to determine congruency and complete the congruence statement. If they are not, answer with NEI (not enough information).

3)

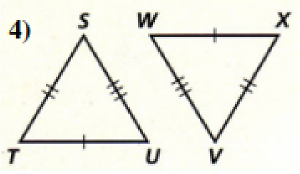


Congruence:

ΔEMN ≅ Δ__________

Reason:

4)

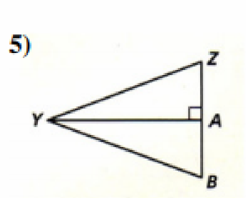


Congruence:

ΔSTU ≅ Δ__________

Reason:

5)

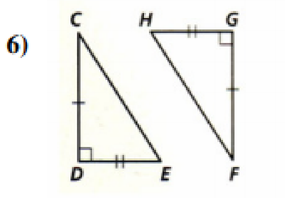


Congruence:

ΔYZA ≅ Δ__________

Reason:

6)



Congruence:

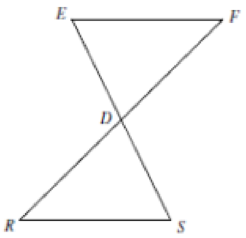
ΔCDE ≅ Δ__________

Reason:

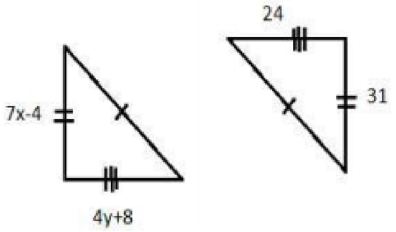
7) Given ΔTUV ≅ ΔGFE, list all congruent parts.

8) Mark the figure to show all congruent parts.

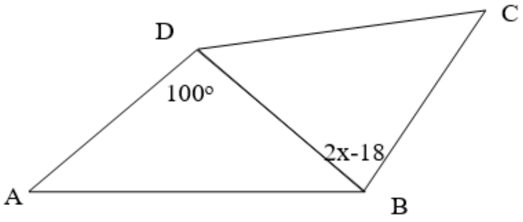
$\Delta DEF \cong \Delta DSR$



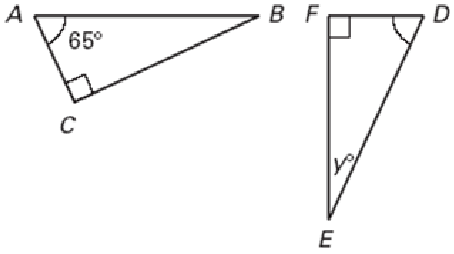
9) Solve for x and y.



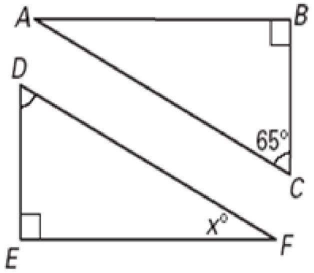
10) If the $m\angle A \cong m\angle C$ and $m\angle ABD \cong m\angle BDC$, find x.



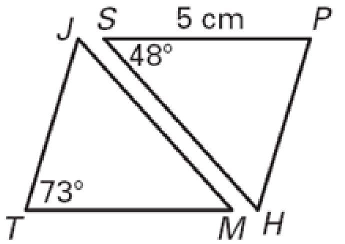
11) Solve for y



12) Find the value of x



In the diagram, $\Delta TJM \cong \Delta PHS$. Complete the statement.



13. $\angle P \cong$ ___?___

14. $\overline{JM} \cong$ ___?___

15. $m\angle M =$ ___?___

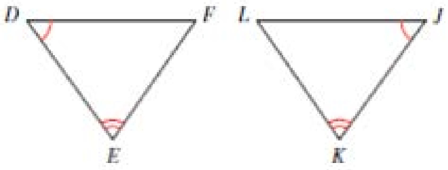
16. $m\angle P =$ ___?___

17. $MT =$ ___?___

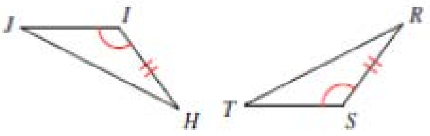
18. $\Delta HPS \cong$ ___?___

For each of the diagrams below, state the additional piece of information to prove the triangles congruent by the given postulate.

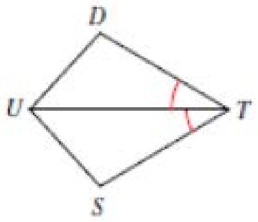
19) ASA



20) SAS

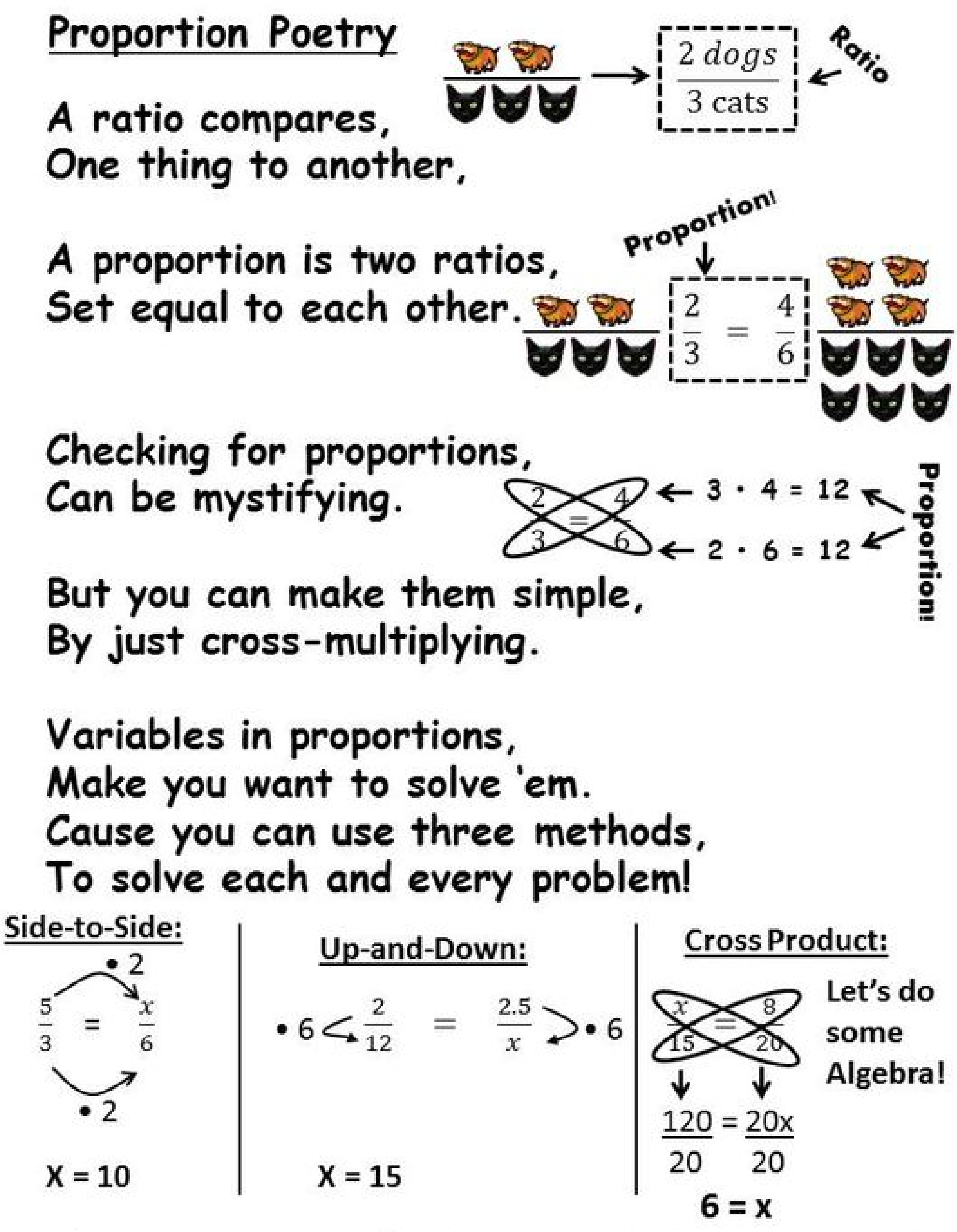


21) AAS



WEEK 3 PART 2 - Similar Triangles Notes

IT ALL STARTS WITH RATIOS AND PROPORTIONS SO LET'S REVIEW WITH A POEM



Proportion Poetry

$\frac{2\ dogs}{3\ \text{cats}}$ ← Ratio

A ratio compares,
One thing to another,

Proportion!
$\frac{2}{3} = \frac{4}{6}$

A proportion is two ratios,
Set equal to each other.

$\frac{2}{3} = \frac{4}{6}$
← 3 · 4 = 12
← 2 · 6 = 12
Proportion!

Checking for proportions,
Can be mystifying.

But you can make them simple,
By just cross-multiplying.

Variables in proportions,
Make you want to solve 'em.
Cause you can use three methods,
To solve each and every problem!

Side-to-Side:

• 2
$\frac{5}{3} = \frac{x}{6}$
• 2

X = 10

Up-and-Down:

$\cdot 6 \quad \frac{2}{12} = \frac{2.5}{x} \quad \cdot 6$

X = 15

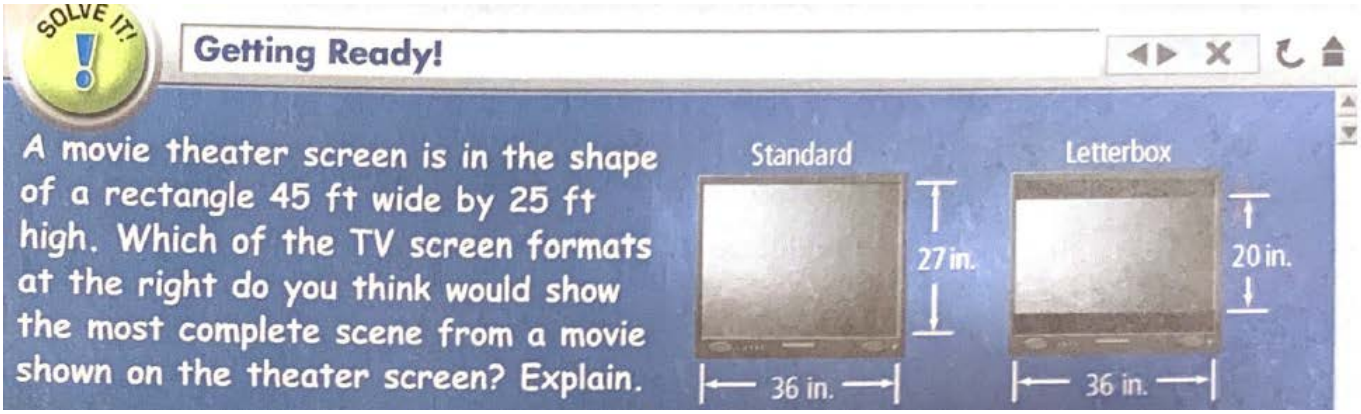
Cross Product:

$\frac{x}{15} = \frac{8}{20}$

Let's do some Algebra!

$\frac{120}{20} = \frac{20x}{20}$

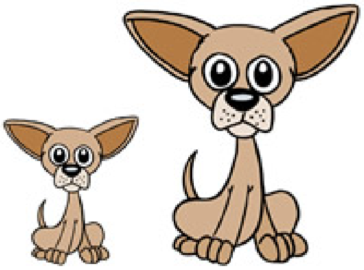
6 = x



A movie theater screen is in the shape of a rectangle 45 ft wide by 25 ft high. Which of the TV screen formats at the right do you think would show the most complete scene from a movie shown on the theater screen? Explain.

Figures with the same shape, but not necessarily the same size, are said to be "similar".

These cartoon dogs are exactly the same shape, but are not the same size.
The dog on the right is an enlargement of the dog on the left.
While these dog figures are not congruent, they are similar.



Definition: Polygons are similar if their corresponding angles are congruent (equal in measure) and the ratios of their corresponding sides are proportional.
(This definition allows for congruent figures to also be "similar", where the ratio of the corresponding sides is 1:1.)

Facts about similar triangles:

Corresponding angles are congruent:

$\angle A \cong \angle D$

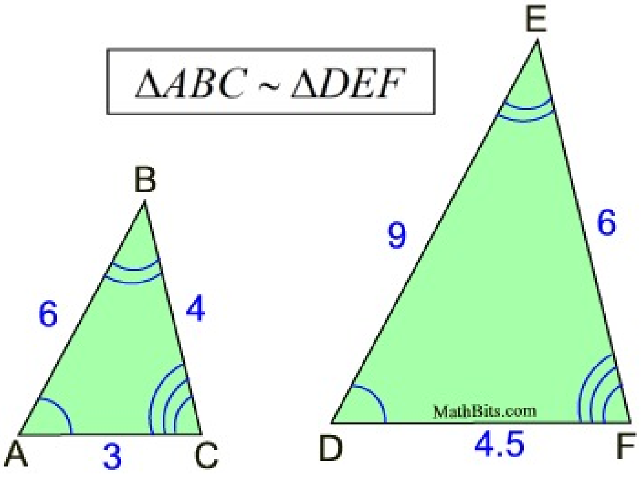
$\angle B \cong \angle E$

$\angle C \cong \angle F$

Corresponding sides are proportional:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$\Delta ABC \sim \Delta DEF$



$$\frac{6}{9} = \frac{4}{6} = \frac{3}{4.5} = \frac{2}{3}$$

The ratio of the corresponding sides is called the ratio of similitude or scale factor.

Similar Symbol: ∼ The symbol used to express "similar" was also seen in the symbol for "congruent" (≅). But unlike congruent, similar does not imply = size.

PROVING TRIANGLES SIMILAR

Reminder: Two triangles are similar if and only if the corresponding sides are in proportion and the corresponding angles are congruent.

Just as there are specific methods for proving triangles congruent (SSS, ASA, SAS, AAS and HL), there are also specific methods that will prove triangles similar.

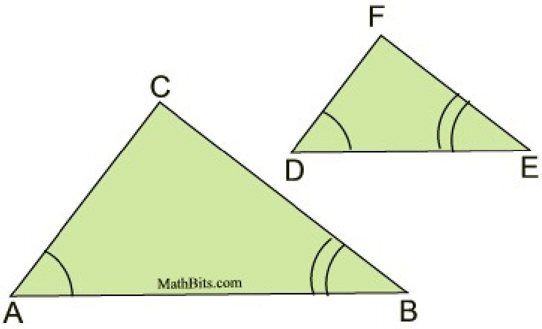
There are three accepted methods for proving triangles similar:

AA To prove two triangles are similar, it is sufficient to show that two angles of one triangle are congruent to the two corresponding angles of the other triangle.

THEOREM: If two angles of one triangle are congruent to the corresponding angles of another triangle, the triangles are similar. (proof of this theorem is shown below)

If: $\angle A \cong \angle D$ and $\angle B \cong \angle E$

Then: $\Delta ABC \sim \Delta DEF$



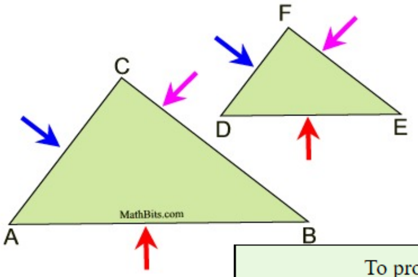
BEWARE The next two methods for proving similar triangles are NOT the same theorems used to prove congruent triangles.

SSS To prove two triangles are similar, it is sufficient to show that the three sets of corresponding sides are in proportion.

THEOREM: If the three sets of corresponding sides of two triangles are in proportion, the triangles are similar.

If: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Then: $\Delta ABC \sim \Delta DEF$



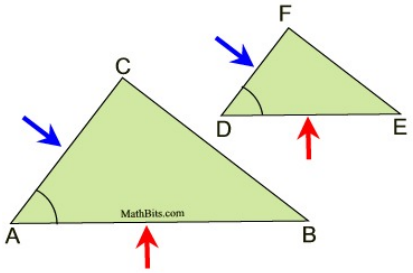
SAS To prove two triangles are similar, it is sufficient to show that two sets of corresponding sides are in proportion and the angles they include are congruent.

THEOREM: If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

If: $\frac{AB}{DE} = \frac{AC}{DF}$

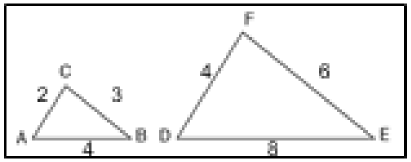
and $\angle A \cong \angle D$

Then: $\Delta ABC \sim \Delta DEF$

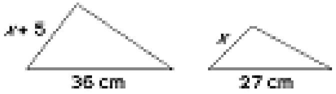


WEEK 3 PART 2 Practice- Similar Triangles

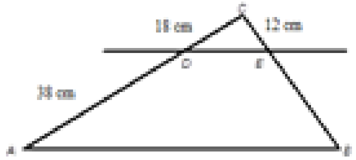
1. $\Delta ACB \sim$ ______



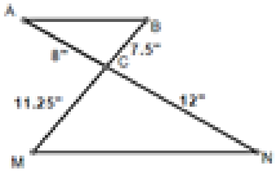
2. The two shapes are similar. Find the missing variable, x = ______



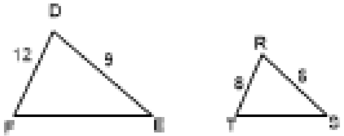
3. In the figure below, $\overline{DE} \parallel \overline{AB}$. To the nearest cm, find BE.



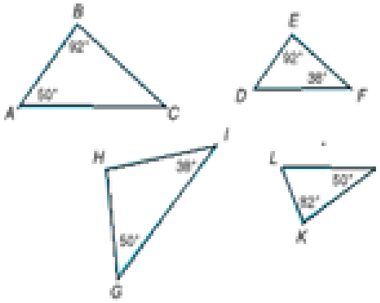
4. $\Delta ABC \sim \Delta NMC$. What is the scale factor of **ΔNMC to ΔABC**?



5. What is needed to prove that $\Delta DEF \sim \Delta RST$, by SAS?

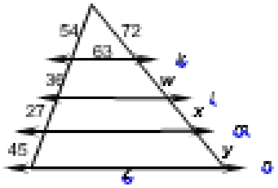


6. Which triangle is NOT similar to the othe



In questions 7 and 8, find the measure of e lettered section shown below.

$k \parallel l \parallel m \parallel n$



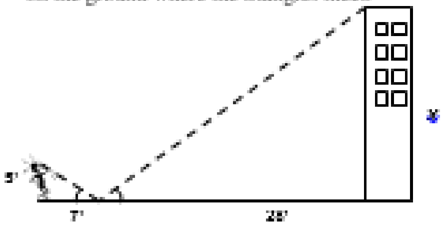
7. w = ______

8. z = ______

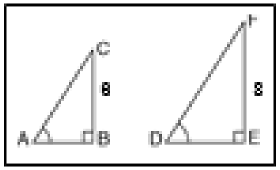
9. If a 12' flagpole casts an 18' shadow at tl time that a nearby building casts a 72' sha How tall is the building?

10. The ratio of angles in a triangle is 3:4:5. is the measure of the smallest angle?

11. Use the diagram below to find the height of the building, assuming that there is a mirror located on the ground where the triangles meet.

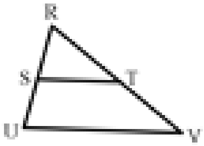


12. Which postulate proves $\Delta ABC \sim \Delta DEF$?



13. Given $\Delta PQM \sim \Delta ZEL$, $PQ = 16$ inches and $ZE = 10$ inches. If the perimeter of ΔPQM is 40 inches, what is the perimeter of ΔZEL?

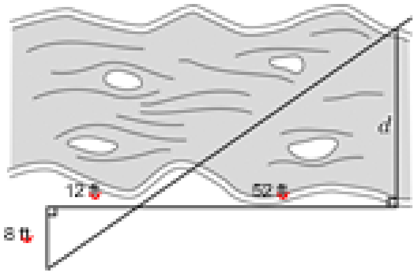
14. Given that $\Delta RST \sim \Delta RUV$, which statement must be true?



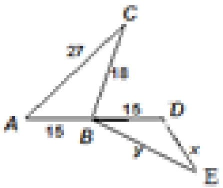
a. $\frac{RS}{RT} = \frac{RU}{UV}$ b. $\frac{RS}{SU} = \frac{ST}{UV}$

c. $\frac{ST}{UV} = \frac{RT}{RV}$ d. $\frac{UV}{ST} = \frac{RS}{RU}$

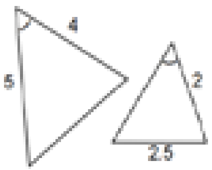
15. The distance across a river was estimated by making the measurements shown. Which is a good estimate of the distance **d**?



16. If $\Delta ABC \sim \Delta EDB$, then y = –?–



17. Which postulate proves the triangles are similar?



18. A volleyball team plays 16 home matches and 12 away matches. What is the ratio of home matches to the total matches played? (Simplify your ratio)

19. Given $\Delta ABE \sim \Delta ACD$

Find the measure of CD.